Glaser & Wolfe

3.2. Find High Temp and Low Temp limits of

$$C = \frac{dU}{dT} = N_A k_B \left(\frac{\theta}{T}\right)^2 \frac{\exp\left(\frac{\theta}{T}\right)}{\left[\exp\left(\theta/T\right) - 1\right]^2}$$

For high T, we approximate $\exp\left(\frac{\theta}{T}\right)$ by

$$1 + \frac{\theta}{T} = \exp\left(\frac{\theta}{T}\right)\Big|_0 + \frac{\frac{d}{d\left(\frac{\theta}{T}\right)}\exp\left(\frac{\theta}{T}\right)\Big|_0 \left(\frac{\theta}{T}\right)}{1!}$$

$$\Rightarrow C \approx N_A k_B \left(\frac{\theta}{T}\right)^2 \frac{\left(1 + \frac{\theta}{T}\right)}{\left(1 + \frac{\theta}{T} - 1\right)^2}$$

$$\approx N_A k_B \left(\frac{\theta}{T}\right)^2 \frac{\left(1 + \frac{\theta}{T}\right)}{\left(\frac{\theta}{T}\right)^2} = N_A k_B \left(1 + \frac{\theta}{T}\right).$$

For small T, $\exp\left(\frac{\theta}{T}\right) - 1 \approx \exp\left(\frac{\theta}{T}\right)$

$$\Rightarrow C \approx N_A k_B \left(\frac{\theta}{T}\right)^2 \frac{\exp\left(\frac{\theta}{T}\right)}{\left(\exp\left(\frac{\theta}{T}\right)\right)^2}$$

$$= \frac{N_A k_B \left(\frac{\theta}{T}\right)^2}{\exp\left(\frac{\theta}{T}\right)}.$$